

# A Binary BBP-type Formula for $\pi\sqrt{5}$

## Kunle Adegoke, May 20, 2012

$$\pi\sqrt{5} = \frac{5}{2^{19}}P(1, 2^{20}, 40, (2^{19}, 2^{19}, -2^{18}, 0, 0, 2^{17}, 2^{16}, 0, 2^{15}, 0, 2^{14}, 0, 2^{13}, 2^{13}, 0, 0, -2^{11}, 2^{11}, 2^{10}, 0, -2^9, -2^9, 2^8, 0, 0, -2^7, -2^6, 0, -2^5, 0, -2^4, 0, -2^3, -2^3, 0, 0, 2, -2, -1, 0))$$

### Proof

The formula follows immediately from the identity:

$$\begin{aligned} \pi = & 2 \operatorname{Im} \operatorname{Li}_1 \left[ \frac{1}{\sqrt{2}} \exp \left( \frac{i\pi}{20} \right) \right] + 2 \operatorname{Im} \operatorname{Li}_1 \left[ \frac{1}{\sqrt{2}} \exp \left( \frac{7i\pi}{20} \right) \right] \\ & + 2 \operatorname{Im} \operatorname{Li}_1 \left[ \frac{1}{\sqrt{2}} \exp \left( \frac{9i\pi}{20} \right) \right] - 2 \operatorname{Im} \operatorname{Li}_1 \left[ \frac{1}{\sqrt{2}} \exp \left( \frac{17i\pi}{20} \right) \right] \end{aligned}$$

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Pi*sqrt(5)=5/2^19*P(1, 2^20, 40,
(2^19, 2^19, -2^18,
0, 0, 2^17, 2^16, 0, 2^15, 0, 2^14,
0, 2^13, 2^13, 0, 0, -2^11, 2^11,
2^10, 0, -2^9, -2^9, 2^8, 0, 0, -2^7,
-2^6, 0, -2^5, 0, -2^4, 0, -2^3,
-2^3, 0, 0, 2, -2, -1, 0))
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